Effects of Kaluza-Klein Excitations on $g_\mu - 2$

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Abstract

An analysis of the effects associated with the Kaluza-Klein excitations of the photon and of the W and Z bosons on $g_\mu - 2$ for d number of extra dimensions with large radius compactifications is given. The Kaluza-Klein effects on $g_\mu - 2$ are found to be very sensitive to the number of extra dimensions. For models where the quark-leptons generations live on the 4D wall, it is shown that when the constraints from the Kaluza-Klein corrections to the Fermi constant are included, the effects of the Kaluza-Klein excitations to $g_\mu - 2$ become too small to be observable. A model which evades Kaluza-Klein corrections to the muon decay $\mu \to e\bar{\nu}_e\nu_\mu$ without suppression of their effects on $g_\mu - 2$ is discussed.

1. Introduction

The anomalous magnetic moment of the muon (g_μ-2) is one of the most accurately determined quantities[1] in particle physics. Here we analyse the effects of Kaluza-Klein[2] excitations on g_μ in theories with large radius compactifications[3−5]. Such theories are currently being investigated because they might arise in the strong coupling limit of the heterotic string which can yield a string scale in the TeV region[3]. The framework of the model we work in is discussed in ref.[5]. Our view point is that the fields of the minimal supersymmetric standard model reside in (4+d) dimensions while gravity propagates in the 10 dimensional bulk. In the language of Type I string the MSSM fields reside on p=3+d branes and we consider

compactifications internal to the brane.

Our analysis is in the framework of an effective field theory and we illustrate our procedure for the Standard Model (SM) case for d=5 . Here our SM fields will consist of gauge fields A_M (M=0,1,2,3,5), the Higgs multiplet H, and the quarks and lepton multiplets. We assume a form of V(H) such that H develops a vacuum expectation value and spontaneous breaking of the electro-weak symmetry takes place so that $H = \frac{1}{\sqrt{2}}(V + H_1 + iH_2)$. We next choose a five dimensional gauge fixing term of the form

$$L_5^{gf} = -\frac{1}{2\xi}(\partial_M A^M - \xi V g_5 H_2)^2 \tag{1}$$

In this gauge the bilinear term involving A_M and H_2 form a total divergence of the $\partial_M(H_2 A^M)$ and hence can be dropped. The remaining terms in the Lagrangian have a simple decomposition among the vector bosons, the Higgs and the fictitious Goldstone bosons. After spontaneous breaking we compactify the Lagrangian on S^1/Z_2 with a radius of compactification R.

We shall assume that the Higgs and the gauge fields live in the five dimensional bulk and the fermions live at one of the orbifold points. Thus we will have only zero modes for the fermion fields in this case. For the 4D Lagrangian we shall work in the $\xi = 1$ gauge. In this gauge, terms bilinear in $A_\mu (\mu = 0, 1, 2, 3)$ and A_5 form a total divergence in 4 dimensions and the A_μ and A_5 decouple at the bilinear level. To normalize the 4D Lagrangian a redefinition of the couplings is needed, i.e., $g_5/\sqrt{\pi R} = g$. Normal mode decomposition on S^1/Z_2 indicates mass terms for the W bosons of $m_W^2 + n^2/R^2$, n=0,1,2,..,∞, where the first term arises from spontaneous breaking of the electro-weak symmetry and the second term arises from the compactification, and similar relations hold for the Kaluza-Klein excitations of the Z boson and for the Higgs boson. The above analysis can be extended to include supersymmetry. Before compactification this 5D theory has an N=2 supersymmetry. After compactification the massless sector of the theory has N=1 supersymmetry while the massive Kaluza-Klein states maintain an N=2 supersymmetry. The model can be extended to d dimensions.

2. Contribution of W and Z Kaluza-Klein Excitations

The framework of the analysis is as in ref.[5]. The analysis of the one loop contribution to $g_\mu-2$ can be carried out in the usual fashion and a finite contribution arises from each Kaluza-Klein mode. Neglecting the relatively small contributions from the Yukawa couplings, the contributions to $g_\mu - 2$ at the one loop level from the W and Z exchange and from their Kaluza-Klein excitations for d extra dimensions is given by

$$(\Delta a)_\mu = \frac{G_F^{SM} m_\mu^2}{\pi^2 2\sqrt{2}}\left(\frac{5}{6}K_d\left(\frac{M_W}{M_R}\right) + \left(-\frac{5}{12} + \frac{4}{3}\left(sin^2\theta_W - \frac{1}{4}\right)^2\right)K_d\left(\frac{M_Z}{M_R}\right)\right) \tag{2}$$

Here a_μ=(g_μ-2)/2, G_F^{SM} is the Fermi constant as evaluated in the Standard Model, θ_W is the weak angle, M_W (M_Z) is the W (Z) boson mass, $M_R = 1/R$ is the common mass scale at which the extra dimensions open up, and the function $K_d(c)$ is defined by

$$K_d(c) = \int_0^\infty dt e^{-t}\left(\theta_3\left(\frac{it}{c\pi}\right)\right)^d \tag{3}$$

Here $\theta_3(\tau)$ for complex τ is given by $\theta_3(\tau)= \sum_{k=-\infty}^{\infty} exp(i\pi k^2\tau)$ where $(Im\tau > 0)$. In the limit c=0 one has $K_d(0) = 1$ and Eq.(2) reduces to the Standard Model result[6]. For the case d=1 Eq.(3) gives a convergent result. However, a cuttoff Λ_d on the lower limit of the integral in Eq.(3) is needed for the case $d \geq 2$. As discussed in Ref.[5] this cutoff is determined consistently by comparison with the truncation on the sum over the Kaluza-Klein states so that only the states up to the string scale are retained in the sum. One finds the following results for Λ_d for $d \geq 2$.

$$\Lambda_2 = \left(\frac{M_R}{M_{str}}\right)^2$$
$$\Lambda_d = \left(\Gamma\left(\frac{d}{2}\right)\right)^{\frac{2}{d-2}}\left(\frac{M_W}{M_{str}}\right)^2, \quad d \geq 3 \tag{4}$$

As discussed in Ref.[5] one must carry out a redefinition of the Fermi constant because of the dressing of the Kaluza-Klein states and the experimentally observed G_F is to be identified as

$$G_F = G_F^{SM} K_d\left(\frac{M_W^2}{M_R^2}\right) \tag{5}$$

Using this redefinition we find the contribution of the Kaluza-Klein modes to a_μ is

$$(\Delta a)_\mu^{W-ZKK} = \frac{G_F m_\mu^2}{\pi^2 2\sqrt{2}}(-\frac{5}{12} + \frac{4}{3}(sin^2\theta_W - \frac{1}{4})^2)(K_d(\frac{M_Z}{M_R}) - K_d(\frac{M_W}{M_R}))/K_d(\frac{M_W}{M_R})) \tag{6}$$

For d=1 $K_1(c)$ is given approximately by $K_1(c) \simeq 1 + \frac{\pi^2}{3}c$ while the $d \geq 2$ cases require a cutoff as discussed above and one finds that the following results give a good approximation

$$K_2(c) = 1 + c(\frac{2\pi^2}{3} + 2\pi ln\frac{M_{str}}{M_R})$$

$$K_d(c)(d \geq 3) = 1 + c(\frac{d}{d-2})\frac{\pi^{d/2}}{\Gamma(1+\frac{d}{2})}(\frac{M_{str}}{M_R})^{d-2} \tag{7}$$

3. Contribution of Photonic Kaluza-Klein Excitations

In the above we computed the contributions arising from the W and Z Kaluza-Klein excitations to a_μ. There are also contributions to a_μ arising from the exchange of the Kaluza-Klein excitations of the photon. Here our analysis gives for the d extra dimensions the result

$$(\Delta a)_\mu^{\gamma KK} = \frac{\alpha}{3\pi}\frac{m_\mu^2}{M_R^2}G \tag{8}$$

where

$$G = \int_0^\infty dt(\theta_3(\frac{it}{\pi}))^d - 1 \tag{9}$$

As for the case of the W and Z Kaluza-Klein exchange contributions, for d=1 the photonic Kaluza-Klein exchange contribution is finite while a cutoff is needed for higher values of d. The cutoff is introduced in the same way as discussed for the case K_d and one gets the following results for various cases

$$(\Delta a)_\mu^{\gamma KK}(d=1) = \alpha\frac{\pi}{9}\frac{m_\mu^2}{M_R^2} \tag{10}$$

$$(\Delta a)_\mu^{\gamma KK}(d=2) = \frac{\alpha}{3\pi}(\frac{2\pi^2}{3} + 2\pi ln\frac{M_{str}}{M_R})\frac{m_\mu^2}{M_R^2} \tag{11}$$

$$(\Delta a)_\mu^{\gamma KK}(d \geq 3) = \frac{\alpha}{3\pi}(\frac{d}{d-2})\frac{\pi^{d/2}}{\Gamma(1+\frac{d}{2})}(\frac{M_{str}}{M_R})^{d-2}\frac{m_\mu^2}{M_R^2} \tag{12}$$

The total Kaluza-Klein exchange contribution to a_μ is the sum of the contribution from the W and Z Kaluza-Klein exchange contribution and from the photonic W and Z exchange contribution

$$a_\mu^{KK} = a_\mu^{W-ZKK} + a_\mu^{\gamma KK} \tag{13}$$

We note that after redefinition of the Fermi constant the W and Z Kaluza-Klein exchange contribution a_μ^{W-ZKK} as given by Eq.(6) is negative while the photonic Kaluza-Klein exchange contribution $a_\mu^{\gamma KK}$ as given by Eq.(8) is positive. Thus one has a partial cancellation between these two contributions. We emphasize here that in any numerical analysis of Eq.(13) the constraints arising from Eq.(5) (which holds only within the current error bars of G_F^{SM} and experiment on G_F) should be included. For the compactifications considered here and in Ref.[5] these constraints on M_R at the 2σ level are $M_R > 1.6$ TeV[5,7] for d=1, $M_R > 3.5$ TeV for d=2, $M_R > 5.7$ TeV for d=3 and $M_R > 7.8$ TeV for d=4.

4. Sizes of Kaluza-Klein Effects on $(g_\mu - 2)$

We discuss now the prospects for the observation of the Kaluza-Klein contributions in g_μ experiment. For this purpose we review first the current situation on a_μ. The current experimental value of a_μ is [1]: $a_\mu^{exp} = 11659230(84) \times 10^{-10}$, where the quantity in the parenthesis is the uncertainty while the corresponding uncertainty in the theoretical determinations is significantly smaller. The most recent Standard model prediction for a_μ is $a_\mu^{theory}(SM) = 11659162.8(6.5) \times 10^{-10}$. This result includes α^5 QED contributions[8], the hadronic vacuum polarization[9] and light by light hadronic contributions[10], and the Standard Model electro-weak contributions[11]. Essentially all of the uncertainty in the Standard Model result arises from the hadronic contributions which include the hadronic vacuum polarization[9], and the light by light hadronic contribution[10]. The total Standard Model electro-weak contribution up to two loops[11] is given by $a_\mu^{EW}(SM) = 15.1(0.4) \times 10^{-10}$.

The new g_μ Brookhaven experiment E821[12] will be able to reduce the current experimental uncertainty by a factor of 20 to a level of 4×10^{-10}. With this sensitivity the new

g_μ experiment will be able to test the Standard Model electro-weak contribution even with the current level of uncertainty in the hadronic contributions. Infact the experiment will be able to probe a_μ to a level of $\sim 8 \times 10^{-10}$ (where we have combined the experimental error and the hadronic error in quadrature). Further, it is expected that the uncertainty in the hadronic error may reduce even more by perhaps as much as a factor of 2 from the data from the ongoing and future precision low energy experiments at VEPP-2M, DAΦNE and BEPC. Of course a further reduction of the hadronic error will sharpen the ability of the Brookhaven experiment to probe new physics.

In the analysis of a_μ^{KK} we impose the gauge coupling unification to constrain the ratio M_{str}/M_R the details of which are discussed in ref.[5]. The results are displayed in Fig.1. The analysis exhibits a sharp dependence of the contribution of the Kaluza-Klein excitations to a_μ^{KK} on the number of extra space time dimensions. The sharp dependence on d arises from the summation over the number of Kaluza-Klein states because of the combinatorics factors. As discussed already there is a partial cancellation between the W and Z Kaluza-Klein exchange contribution and the photonic Kaluza-Klein exchange contribution after a redefinition of the Fermi constant is taken into account. As discussed above and in Ref.[5] the analysis of the Kaluza-Klein mode contribution to G_F gives strong constraints on M_R. Fig.1 shows that with these constraints on M_R the new g_μ experiment will not come even close to exploring the extra dimensions, ie., the total Kaluza-Klein contribution falls more than 1-2 orders of magnitude below the sensitivity that will be achievable in the new g_μ experiment.

In the above we assumed that all the quark-lepton generations lie on the 4-dimensional wall. We discuss now a variant of the model considered above where the first quark-lepton generation lies on the wall while the second quark-lepton generation lives in the bulk. In this case the first quark-lepton generation will have no couplings with the Kaluza-Klein modes while the second generation will have such couplings. In this model the process $\mu \to e\bar{\nu}_e\nu_\mu$ receives no Kaluza-Klein correction at the tree level and thus G_F is uncorrected at this level. Similarly in this case the atomic parity violating interactions receive no Kaluza-Klein corrections. Consequently there are no constraints arising at the tree level from the

experimental accuracy of G_F and from the atomic parity experiments on M_R, although corrections at the loop level can arise which are, however, expected to be much smaller than what one would otherwise expect. Other experimental bounds as those arising from contact interactions using LEP data are also evaded in this model. Further, one would also expect here a significant suppression of the production of the Kaluza-Klein excitations of the W and the Z boson at the $\bar{p}p$ colliders due to the dominance of the first generation in the quark content of p($\bar{p}$). However, Kaluza-Klein corrections to $g_\mu - 2$ in this model are not suppressed and the analysis of a_μ^{KK} in this case is given in Fig.2 again under the unification of the gauge coupling constant constraint. (The analysis uses Eq.2 with the measured value of G_F^{SM}). The analysis of Fig.2 shows that the new g_μ experiment will be able to probe two extra dimensions up to scales $M_R \sim 0.65$ TeV, three extra dimensions up to scales $M_R \sim 1$ TeV, and four extra dimensions up to scales $M_R \sim 1.4$ TeV. We note that the scales which can be explored well exceed the mass bound on the Kaluza-Klein modes of the quarks inferred from the fourth generation quark searches which gives a lower limit of 130 GeV[13].

5. Conclusion

In this paper we have investigated the effects of Kaluza-Klein excitations arising from extra dimensions with large radius compactifications on $g_\mu - 2$. These effects consist of the photonic Kaluza-Klein exchange contribution and of the W and Z exchange contribution. For the case of one extra dimension the Kaluza-Klein contribution to a_μ is finite. However, a cutoff is necessary for the case of d extra dimensions, $d \geq 2$. We have derived approximate relations for the Kaluza-Klein contributions for these cases (see Eqs. 6,7,10-12). For d=2 these approximate relations are accurate to $O(10\%)$ while for $d \geq 3$ the relations are accurate to $O(1-2\%)$. We have given a quantitative analysis of the contributions of the Kaluza-Klein excitations of γ, W and Z under the constraints of the unification of the gauge coupling constants. Our analysis shows that for the case when all the quark-lepton generations lie on the 4D wall, inclusion of the constraints on M_R arising from the analysis of Kaluza-Klein

mode contributions to G_F lead to effects on $g_\mu - 2$ which are too small to be observable by the new g_μ experiment. We also considered a model where the first generation lives in the bulk and the second generation lies on the wall. In this case effects from extra dimensions $d \geq 2$ may become visible in the Brookhaven experiment. Furthermore, a muon collider may be able to explore these extra dimensions directly. Of course a fundamental string model will have to justify such an assignment of generations between the bulk and the wall.

After completion of this work there appeared a paper[14] where an analysis of the effects of Kaluza-Klein excitations of gauge bosons on $g_\mu - 2$ is also given. However, the constraints arising from the effects of Kaluza-Klein corrections to the Fermi constant are not included.

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FIGURES

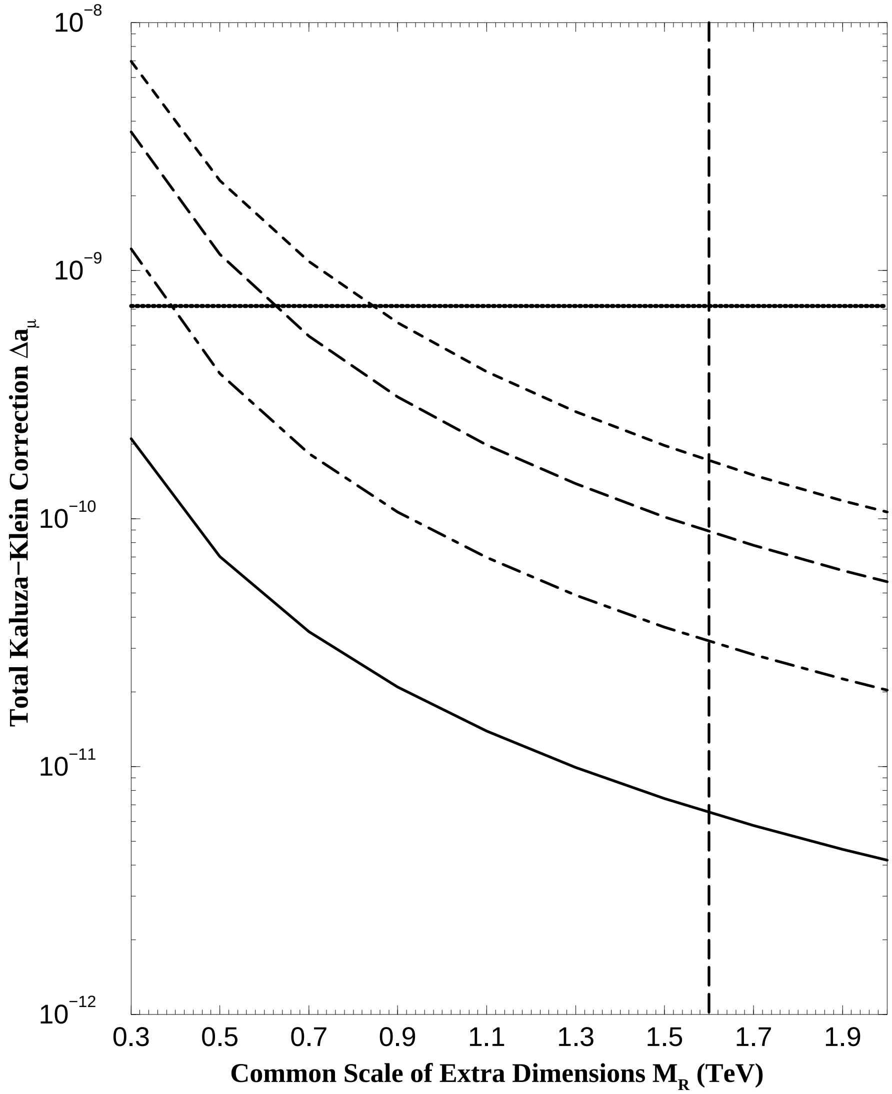

Fig. 1. Plot of sum of the photonic Kaluza-Klein exchange contribution and the W and Z Kaluza-Klein exchange contribution to a_μ as a function of the common scale of extra dimension when the G_F constraint is included. The curves correspond to the case d=1 (solid), d=2 (dot-dashed), d=3 (long dashed), and d=4 (dashed). The horizontal dotted line corresponds to the 1σ limit to which the Brookhaven experiment E821 will be able to probe a_μ. The G_F constraint requires $M_R > 1.6$ TeV for d=1 and the allowed region lies to the right of the vertical dashed curve. The constraints on M_R for $d > 1$ are even more severe.

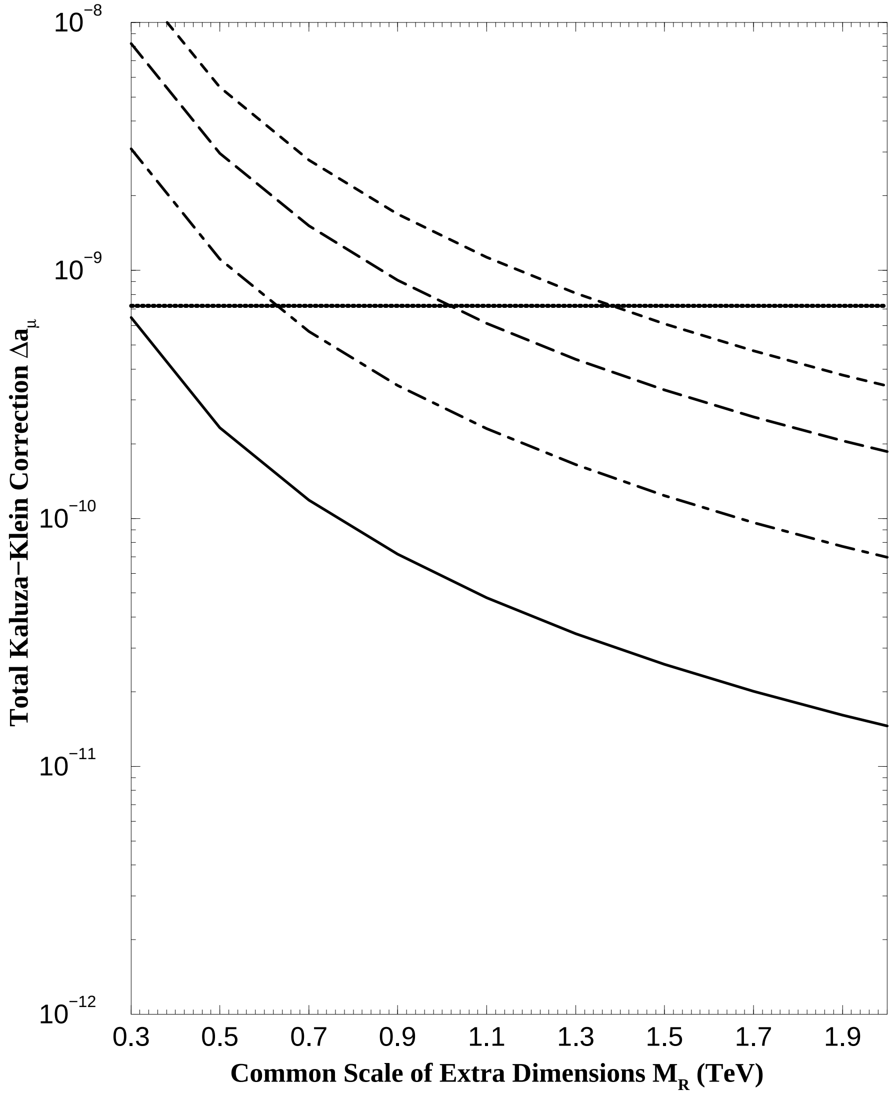

Fig. 2. Plot of sum of photonic Kaluza-Klein exchange contribution and the W and Z Kaluza-Klein exchange contribution to a_μ as a function of the common scale of extra dimension when there are no Kaluza-Klein corrections to G_F. The curves correspond to the case d=1 (solid), d=2 (dot-dashed), d=3 (long dashed), and d=4 (dashed). The horizontal dotted line is the same as in Fig.1.